

Trigonometrische Funktion - Sonnenschein

$$f(x) = 3 \sin\left(\frac{\pi}{4}x\right) \quad x \in [0; 8]$$

$$\begin{aligned} f'(x) &= 3 \cos\left(\frac{\pi}{4}x\right) \left(\frac{\pi}{4}\right) \\ &= \frac{3\pi}{4} \cos\left(\frac{\pi}{4}x\right) \end{aligned}$$

$$\begin{aligned} f''(x) &= -\frac{3\pi^2}{4} \sin\left(\frac{\pi}{4}x\right) \left(\frac{\pi}{4}\right) \\ &= -\frac{3\pi^3}{16} \sin\left(\frac{\pi}{4}x\right) \end{aligned}$$

$$\begin{aligned} f'''(x) &= -\frac{3\pi^4}{16} \cos\left(\frac{\pi}{4}x\right) \left(\frac{\pi}{4}\right) \\ &= -\frac{3\pi^5}{64} \cos\left(\frac{\pi}{4}x\right) \end{aligned}$$

Extrempunkte

n. B.: $f'(x) = 0$

$$0 = \frac{3\pi}{4} \cdot \cos\left(\frac{\pi}{4}x\right)$$

$$0 = \cos\left(\frac{\pi}{4}x\right)$$

$$\frac{\pi}{4}x = \pi$$

$$\cos(\pi) = 0$$

$$\pi = \frac{\pi}{2} \quad \vee \quad \pi = \frac{3\pi}{2}$$

$$\frac{\pi}{4}x = \frac{\pi}{2}$$

$$\frac{\pi}{4}x = \frac{3\pi}{2}$$

$$x = 2$$

$$\vee \quad x = 6$$

n. B.: $f''(x) \neq 0$

$$f''(2) = -\frac{3\pi^3}{16} \sin\left(\frac{\pi}{2}\right)$$

$$= -\frac{3\pi^3}{16} \pi^2 (1) < 0 \quad \Rightarrow \text{Max}$$

$$f(2) = 3 \sin\left(\frac{\pi}{2}\right) = 3$$

HP(2|3)

$$f''(6) = -\frac{3}{16}\pi^2 \sin\left(\frac{3\pi}{2}\right)$$

$$= -\frac{3}{16}\pi^2 (-1) > 0 \Rightarrow \text{Min}$$

$$f(6) = 3 \sin\left(\frac{3\pi}{2}\right) = -3$$

TP(6|-3)

Wendepunkte

n. B.: $f''(x) = 0$

$$0 = -\frac{3\pi^2}{16} \sin\left(\frac{\pi}{4}x\right)$$

$$0 = \sin\left(\frac{\pi}{4}x\right)$$

$$\frac{\pi}{4}x = r$$

$$\sin(r) = 0$$

$$r = 0 \vee r = \pi \vee r = 2\pi$$

$$\frac{\pi}{4}x = 0 \quad \frac{\pi}{4}x = \pi \quad \frac{\pi}{4}x = 2\pi$$

$$x = 0 \vee x = 4 \vee x = 8$$

n. B.: $f'''(x) \neq 0$

$$f'''(0) = -\frac{3\pi}{8} \pi^3 \cos\left(\frac{\pi}{4} \cdot 0\right) \neq 0$$

$$f'''(4) = -\frac{3\pi}{8} \pi^3 \cos\left(\frac{\pi}{4} \cdot 4\right) \neq 0$$

$$f'''(8) = -\frac{3\pi}{8} \pi^3 \cos\left(\frac{\pi}{4} \cdot 8\right) \neq 0$$

$$f(0) = 3 \sin(0) = 0$$

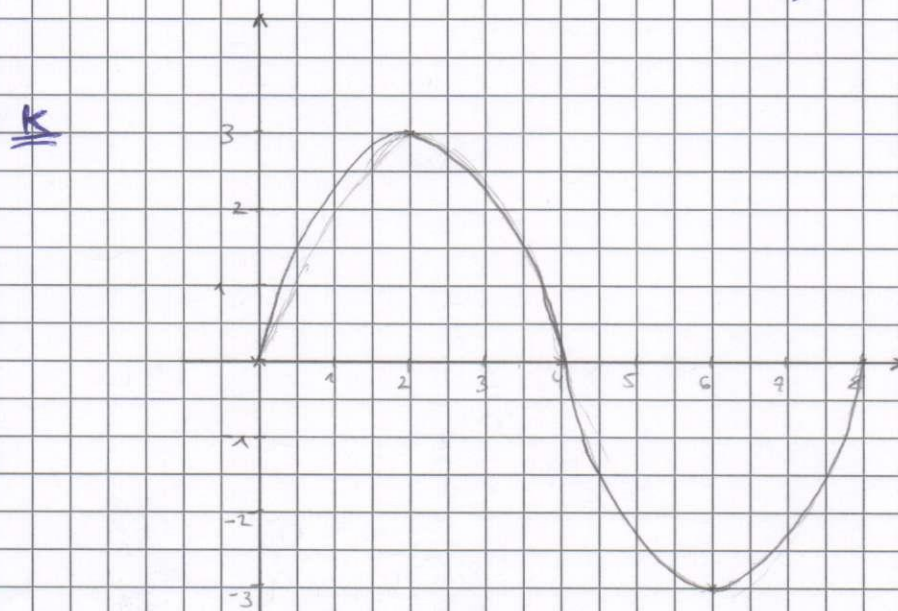
$$\text{WP}_1 (0|0)$$

$$f(4) = 3 \sin(\pi) = 0$$

$$\text{WP}_2 (4|0)$$

$$f(8) = 3 \sin(2\pi) = 0$$

$$\text{WP}_3 (8|0)$$



Volumen

$$3 \sin\left(\frac{\pi}{4}x\right) = 0$$

$$\sin\left(\frac{\pi}{4}x\right) = 0$$

⇒ s.o.

$$x=0 \quad \vee \quad x=4 \quad \vee \quad x=8$$

$$\begin{aligned} V &= \left| \int_0^4 3 \sin\left(\frac{\pi}{4}x\right) dx \right| + \left| \int_4^8 3 \sin\left(\frac{\pi}{4}x\right) dx \right| \\ &= \left| \left[-\frac{12}{\pi} \cos\left(\frac{\pi}{4}x\right) \right]_0^4 \right| + \left| \left[-\frac{12}{\pi} \cos\left(\frac{\pi}{4}x\right) \right]_4^8 \right| \\ &= \left| -\frac{12}{\pi}(-1) + \frac{12}{\pi}(1) \right| + \left| -\frac{12}{\pi}(1) + \frac{12}{\pi}(-1) \right| \\ &= \left| \frac{12}{\pi} + \frac{12}{\pi} \right| + \left| -\frac{12}{\pi} - \frac{12}{\pi} \right| \\ &= \frac{24}{\pi} + \frac{24}{\pi} \\ &= \frac{48}{\pi} \approx 15,28 \end{aligned}$$

b)

$$S(t) = a + b \cdot \sin\left(\frac{\pi}{6}t\right) \quad a, b \in \mathbb{R}$$

8 März: $S(-1) = 100$

Maï: $S(1) = 200$

$$S(1) = a + b \cdot \sin\left(\frac{\pi}{6}(1)\right)$$

$$= a + \frac{1}{2}b$$

$$200 = a + \frac{1}{2}b$$

$$a = 200 - \frac{1}{2}b$$

$$S(-1) = a + b \cdot \sin\left(-\frac{\pi}{6}\right)$$

$$= a - \frac{1}{2}b$$

$$100 = a - \frac{1}{2}b$$

$$100 = \left(200 - \frac{1}{2}b\right) - \frac{1}{2}b$$

$$100 = 200 - b$$

$$b = 100$$

$$a = 200 - \frac{1}{2}b$$

$$a = 200 - 50$$

$$a = 150$$

Oktober: $t = 6$

$$S(6) = 150 + 100 \cdot \sin(\pi)$$

$$= 150 + 100(0)$$

$$= 150$$

Die Sonnenscheindauer im Oktober beträgt 150 Sonnenstunden.

→ 6 Stunden Abweichung

$$\frac{156}{100} \cdot 6 = 9,36\%$$

$$\begin{aligned} S'(t) &= 100 \cos\left(\frac{\pi}{6}t\right) \frac{\pi}{6} \\ &= \frac{50}{3}\pi \cos\left(\frac{\pi}{6}t\right) \end{aligned}$$

$$S'(t) = 0$$

$$0 = \frac{50}{3}\pi \cos\left(\frac{\pi}{6}t\right)$$

$$0 = \cos\left(\frac{\pi}{6}t\right)$$

$$\frac{\pi}{2} = \frac{\pi}{6}t \quad \vee \quad \frac{3\pi}{2} = \frac{\pi}{6}t$$

$$t = 3 \quad \vee \quad t = 9$$

$$S''(t) = -\frac{50}{18}\pi^2 \cdot \sin\left(\frac{\pi}{6}t\right)$$

$$S''(3) = -\frac{50}{18}\pi^2 (1) < 0 \Rightarrow \text{Max}$$

$$S''(9) = -\frac{50}{18}\pi^2 (-1) > 0 \Rightarrow \text{Min}$$

→ kleinster y-Wert 0, da Stunden nicht negativ sein können

⇒ nur das Max. ist entscheidend

$$S(3) = 150 + 100 \sin\left(\frac{\pi}{2}\right)$$

$$= 150 + 100 (1)$$

$$= 250$$

$$W = [0; 250]$$

Sonnenscheindauer > 235 Stunden

$$235 = 150 + 100 \cdot \sin\left(\frac{\pi}{6}t\right)$$

$$85 = 100 \cdot \sin\left(\frac{\pi}{6}t\right)$$

$$0,85 = \sin\left(\frac{\pi}{6}t\right)$$

$$\frac{\pi}{6}t = \frac{\pi}{3} \quad \vee \quad \frac{\pi}{6}t = \frac{2\pi}{3}$$

$$t = 2 \quad \vee \quad t = 4$$

Da das Max. bei $t=3$ liegt scheint die Sonne zwischen Juni und August mehr als 235 Stunden.

durchschnittliche Sonnenscheindauer $\Rightarrow f(c)$

Oktober $\rightarrow t=6$

März $\rightarrow t=11$

$$f(c) = \frac{\int_a^b f(x) dx}{b-a}$$

$$f(c) = \frac{\int_6^{11} S(t) dx}{11-6}$$

$$= \frac{\int_6^{11} (150 + 100 \sin(\frac{\pi}{6}t)) dx}{5}$$

$$= \frac{[150t - \frac{600}{\pi} \cos(\frac{\pi}{6}t)]_6^{11}}{5}$$

$$= \frac{[1650 - \frac{600}{\pi}(-\frac{1}{2})] - [900 - \frac{600}{\pi}(-1)]}{5}$$

$$= \frac{1650 + 300/\pi - 900 - \frac{600}{\pi}}{5}$$

$$= \frac{750 - \frac{300}{\pi}}{5}$$

$$= 150 - \frac{60}{\pi}$$

$$= 130,90 \text{ Stunden}$$

A.3: $S''(t) = 0$

$$S'(t) = \frac{50\pi}{3} \cos\left(\frac{\pi}{6}t\right)$$

$$S''(t) = -\frac{50\pi^2}{18} \sin\left(\frac{\pi}{6}t\right)$$

$$= -\frac{25}{9} \pi^2 \sin\left(\frac{\pi}{6}t\right)$$

$$0 = -\frac{25}{9} \pi \sin\left(\frac{\pi}{6}t\right)$$

$$0 = \sin\left(\frac{\pi}{6}t\right)$$

$$\frac{\pi}{6}t = 0 \vee \frac{\pi}{6}t = \pi \vee \frac{\pi}{6}t = 2\pi \vee \frac{\pi}{6}t = 3\pi$$

$$t = 0 \vee t = 6 \vee t = 12 \vee t = 18$$

↓

April

↓

Oktober

↓

April

↓

Oktober

A.4: $S'''(t) \neq 0$

$$S''(t) = -\frac{25}{9} \pi^3 \cos\left(\frac{\pi}{6}t\right)$$

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$$S'''(0) = -\frac{25}{9} \pi^3 \cos(0)$$

$$= -\frac{25}{9} \pi^3 (1)$$

$\neq 0$

$$S'''(6) = -\frac{25}{9} \pi^3 \cos(\pi)$$

$$= -\frac{25}{9} \pi^3 (-1)$$

$\neq 0$

In den Monaten April und Oktober ändert sich die Sonnenscheindauer am raschesten.