

Solving Math Problems

Layout for Notes

divide your writing sheet into 2 columns

left column: main material

right column: reflections

Get Started

- collect & organize data
- look at simpler cases
- look at special cases
- look at extreme cases
- break down the problem into parts
- check definitions

Find Representations

Basic Ideas

- collect several representations
- start with most promising one

Useful Ideas

- introduce notation
- draw diagrams
 - algebraic
 - b-adic factorisation, primes
- organize data
 - in tables
 - in diagrams...
- exploit symmetries invariants
- geometric
 - coordinate: cartesian, polar, cylindrical...
 - choice of origin
- develop mental images

Basic Ideas

- build a "planning tree" by asking "How can I do this?" several times
- collect several alternative approaches
- start with most promising approach
- use wishful thinking: make the problem nicer
- use forward search
- use backward search

Types of Proof

- direct proof
- induction
- transposition
- contradiction
- construction
- exhaustion
- probabilistic proof
- combinatorial
- nonconstructive
- visual proof
- computer-assisted

Make a Plan & carry it out

Important Principles

- analogy
 - look at similar problems
- induction
 - infinite descent
- guess & check
- extremes
- symmetry
- invariance
 - monovariance
- modularisation
- stepwise approximation
- Fubini Principle
- parity
- The Box Principle
- inclusion & exclusion
- opposites
- generalisation
- specialisation
- variation
- recursion
- colouring
- randomisation
- change of perspective
- brute force
- greedy algorithm
- use a computer
- constant vs. variable
- parameters - change!

Find New Approaches

- basic principle:
 - choose objects
 - modify objects
 - observe results
- what to modify
 - problem objects
 - functions, numbers, sets...
 - unknown
 - data
 - conditions
 - representations
 - plan
- how to modify
 - substitute, replace
 - combine with other elements
 - reverse, rearrange
 - eliminate
 - exchange
 - adapt, alter
 - add
 - minimize, maximize
 - break down into parts
 - approximate
 - combine with other ideas
- what to look at
 - symmetry
 - patterns
 - extremes
 - limits
 - data
 - invariants
 - details - more or less of them
 - parity

Gather & Use Information

- use solved problems
 - use their results
 - use their methods
- use relevant theorems
- use general math concepts
 - complex numbers
 - graphs
 - generating functions
 - ...
- talk to others
 - directly / via mail
- use the internet
 - math encyclopedias
 - math communities
 - accessing literature
- use books
 - scripts, books, formularies

Reflect

reflect on the way

- what is the conflict?
- where is the confusion?
- what can you do?
- do something!
- correctness proved?
- correctness evident?
- working without aim
- working without plan
- errors in carrying out the plan
- lack of reflection
- check lists of errors

reflect at the end

- what worked?
- what didn't work?
- use results elsewhere
- use methods elsewhere

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Persist

- work on for just 15 minutes
- ...and repeat this
- use coping self talk
- remember previous successes
- imagine the work done

Stay Functional

- talk to people
- eat / drink...
- exercise, physical activity
- breathe deeply and calmly
- take a break
- sleep
- work in a new setting
- music - amke or listen
- nonmath activity
- math activity in another domain
- flood yourself with new ideas