3. Abituraufgabe

Exponentialfunktion

2.)

$$f_k(x) = \frac{e^{kx}}{(e^{kx}+1)^2}$$

$g(x) = e^{kx}+1 \qquad g'(x) = e^{kx}\cdot k$

$h(t) = \frac{1}{t^2} \qquad H(t) = {-1}/{t}$

$$\int f_k(x)\,dx = \frac{1}{k}\int h(g(x))\cdot g'(x)\,dx$$
$$= \frac{1}{k}\cdot H(g(x))$$
$$= \frac{1}{k}\left(\frac{-1}{e^{kx}+1}\right)+c$$
$$= \frac{-1}{k(e^{kx}+1)}+c$$
$$= F_k(x)$$

3.) Bsp: $f(x) = x^2$ achsensymmetrisch

$f'(x) = 2x$ punktsymmetrisch

Vor:

$f(-x) = f(x)$

Beh:

$f'(-x) = -f'(x)$

differential Quotient gesucht

Bew:

$$f'(-x) = \lim_{h\to 0}\frac{f(-x+h)-f(-x)}{h}$$
$$= \lim_{h\to 0}\frac{f(x-h)-f(x)}{h}$$
$$= \lim_{h\to 0} -\frac{f(x)-f(x-h)}{h}$$

$$= -\lim_{h \to 0} \frac{f(x) - f(x-h)}{h}$$

$$= -f'(x)$$

q.e.d

4.1)

- $\varphi(x) = \frac{1}{2\sqrt{\pi}} \cdot e^{-0{,}5x^2}$

- $\Phi(t) = \int_{-\infty}^{t} \varphi(x)\, dx$

$$\Rightarrow \int_0^{0{,}5} \varphi(x) = \Phi(0{,}5) - \Phi(0)$$

$$= 0{,}6915 - 0{,}5$$

$$= 0{,}1915$$

- $g(x) = f_{1{,}5}(x) \cdot a$

$$\int_0^{0{,}5} g(x)\, dx = \int_0^{0{,}5} \varphi(x) = 0{,}1915$$

$$\left[\frac{-a}{1{,}5(e^{1{,}5x}+1)}\right]_0^{0{,}5} = 0{,}1915$$

$$-\frac{a}{1{,}5}\left[\frac{1}{e^{1{,}5x}+1}\right]_0^{0{,}5} = 0{,}1915$$

$$-\frac{a}{1{,}5}\left(\frac{1}{e^{0{,}75}+1} - \frac{1}{2}\right) = 0{,}1915$$

$$-\frac{a}{1{,}5}(-0{,}19) = 0{,}1915$$

$$a \approx 1{,}6$$

$$g(x) = \frac{1{,}6 \cdot e^{1{,}5x}}{(e^{1{,}5x}+1)^2}$$

1. Quadrant: von 0 - ∞

$$A = \int_0^{\infty} g(x)\,dx$$

$$= \lim_{z \to \infty} \int_0^z g(x)\,dx$$

$$= \lim_{z \to \infty} \left[\frac{-1{,}6}{1{,}5\,(e^{1{,}5x}+1)} \right]_0^z$$

$$= \frac{-1{,}6}{1{,}5} \lim_{z \to \infty} \left[\frac{1}{e^{1{,}5x}+1} \right]_0^z$$

$$= \frac{-1{,}6}{1{,}5} \lim_{z \to \infty} \Big(\underbrace{\frac{1}{e^{1{,}5z}+1}}_{0} \underbrace{- \frac{1}{2}}_{-1/2} \Big)$$

$$= \frac{-1{,}6}{1{,}5} \left(-\frac{1}{2}\right)$$

$$= 0{,}53$$