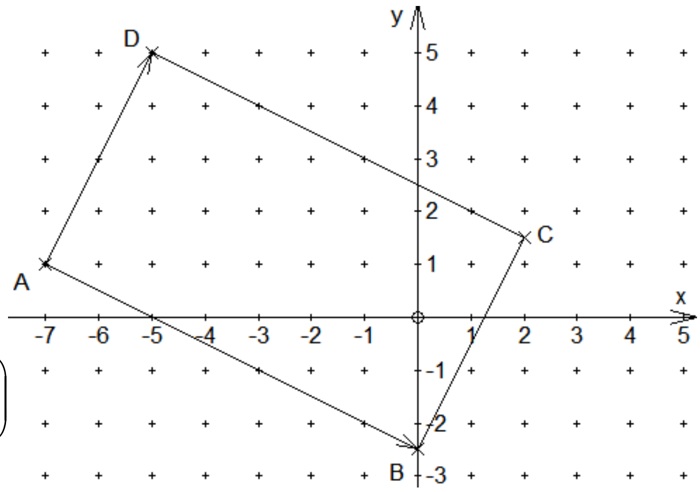
3) A(– 7 | 1); B(0 | y); D(– 5 | 5)

Gemeinsamer Fußpunkt A



$\overrightarrow{AB} = \begin{pmatrix} 0+7 \\ y-1 \end{pmatrix} = \begin{pmatrix} 7 \\ y-1 \end{pmatrix}$ $\qquad \overrightarrow{AD} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\frac{A}{FE} = \begin{vmatrix} 7 & 2 \\ y-1 & 4 \end{vmatrix} = 28 - 2\,(y-1) = 28 - 2y + 2 = 30 - 2y$

$30 - 2y = 35 \quad \Leftrightarrow \quad 2y = -5 \quad \Leftrightarrow \quad y = -2{,}5$

B (0 | – 2,5)

$\overrightarrow{OC} = \overrightarrow{OB} \oplus \overrightarrow{BC} = \overrightarrow{OB} \oplus \overrightarrow{AD} = \begin{pmatrix} 0 \\ -2{,}5 \end{pmatrix} \oplus \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1{,}5 \end{pmatrix}$

C(2 | 1,5)

b) Das Parallelogramm ist offensichtlich ein Rechteck.

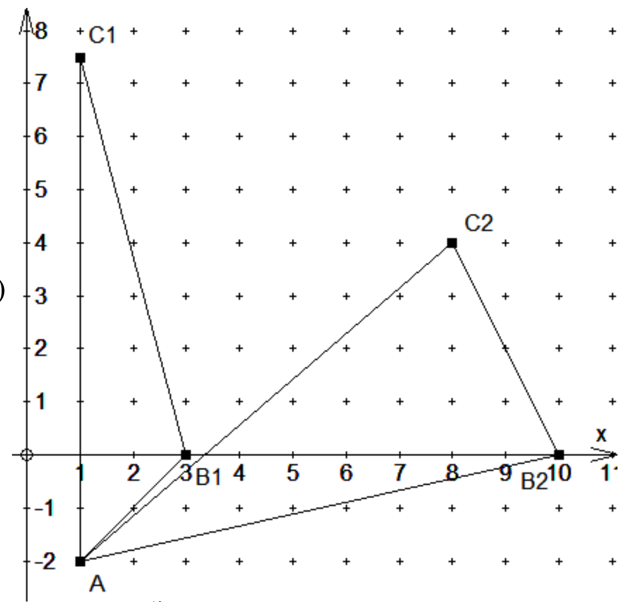
Beweis: Das Produkt der Steigungen von AD und von BC muss – 1 sein.

Steigung von AD: $\quad m = \frac{5-1}{-5+7} = 2$

Steigung von AB: $\quad m = \frac{-2{,}5-1}{0+7} = -0{,}5$

$2 \cdot (-0{,}5) = -1$ $\quad$ was zu beweisen war (w.z.b.w.)

4) A(1 | – 2), B_n(x +2 | 0) und C_n(x | – 0,5x + 8)



a) x = 1 → B_1(1 +2 | 0) = B_1(3 | 0) $\quad$ C_1(1 | – 0,5 + 8) = C_1(1 | 7,5)

x = 8 → B_2(8 +2 | 0) = B_2(10 | 0) $\quad$ C_2(8 | – 0,5·8 + 8) = C_2 (8 | 4)

b) $\overrightarrow{AB_n} = \begin{pmatrix} x+2-1 \\ 0+2 \end{pmatrix} = \begin{pmatrix} x+1 \\ 2 \end{pmatrix}$

$\overrightarrow{AC_n} = \begin{pmatrix} x-1 \\ -0{,}5x+8+2 \end{pmatrix} = \begin{pmatrix} x-1 \\ -0{,}5x+10 \end{pmatrix}$

$\frac{A}{FE} = 0{,}5 \begin{vmatrix} x+1 & x-1 \\ 2 & -0{,}5x+10 \end{vmatrix} = 0{,}5 \cdot [(x+1)(-0{,}5x+10) - 2(x-1)] = 0{,}5 \cdot (-0{,}5x^2 + 10x - 0{,}5x + 10 - 2x + 2) =$

$-0{,}25x^2 + 3{,}75x + 6$

c) AB_2C_2 (da war x = 8) $\quad \frac{A_2}{FE} = -0{,}25 \cdot 8^2 + 3{,}75 \cdot 8 + 6 = 20$

d) $\frac{A}{FE} = -0{,}25x^2 + 3{,}75x + 6$

a = – 0,25 $\qquad$ b = 3,75 $\qquad$ c = 6

x = (– b) : (2 a) = – 3,75 : (– 0,5) = 7,5

$\frac{A}{FE} = -0{,}25 \cdot 7{,}5^2 + 3{,}75 \cdot 7{,}5 + 6 = 20{,}0625$ $\qquad$ ***A_{max} = 20,0625 FE für x = 7,5***

5a) $x^2 = 0{,}64 \quad \Leftrightarrow \quad x = 0{,}8 \;\; v \;\; x = -0{,}8 \qquad$ IL = {– 0,8; 0,8}

b) $x^2 + 9 = 0 \quad \Leftrightarrow \quad x^2 = -9 \qquad$ IL = { }

c) $3x^2 - 27 = 0 \Leftrightarrow 3x^2 = 27 \Leftrightarrow x^2 = 9 \Leftrightarrow x = -3 \; v \; x = 3 \qquad$ IL = {– 3; 3}

d) $x^2 = 50 - x^2 \Leftrightarrow 2x^2 = 50 \quad \Leftrightarrow x^2 = 25 \quad \Leftrightarrow x = -5 \;\; v \;\; x = 5 \qquad$ IL = {– 5; 5}

e) $3{,}63 - x^2 = 2x^2 \Leftrightarrow 3{,}63 = 3x^2 \Leftrightarrow x^2 = 1{,}21 \quad \Leftrightarrow x = -1{,}1 \;\; v \;\; x = 1{,}1 \qquad$ IL = {– 1,1; 1,1}

6a) Die Variablen vertreten positive Zahlen.

b) $\sqrt{625x^3} = 25x\sqrt{x}$ $\qquad$ c) $\sqrt{32b^3} = \sqrt{16b^2 \cdot 2b} = 4b\sqrt{2b}$ $\qquad$ d) $\sqrt{40c^4} = \sqrt{4c^4 \cdot 10} = 2c^2 \cdot \sqrt{10}$

e) $\sqrt{60a^3b^4} = \sqrt{4a^2b^4 \cdot 15a} = 2ab^2\sqrt{15a}$

6b)

a) 4; 5; 11; 21; 19; 15; 14

b) $\sqrt{1000} = \sqrt{100 \cdot 10} = 10 \cdot \sqrt{10}$; 100; $10^3 \cdot \sqrt{10}$; 10 000; 1 000 000

c) 0,1; $\sqrt{0{,}001} = \sqrt{0{,}0001 \cdot 10} = 0{,}01 \cdot \sqrt{10}$; $0{,}01 = 10^{-2}$; $0{,}0001 = 10^{-4}$; 10^{-6}

7a) $\dfrac{\sqrt{125u^5z^3}}{\sqrt{625u^3z^5}} = \dfrac{5u^2z\sqrt{5uz}}{25uz^2\sqrt{uz}} = \dfrac{u\sqrt{5}\cdot\sqrt{uz}}{5z\sqrt{uz}} = \dfrac{u\sqrt{5}}{5z}$

$\sqrt{\dfrac{1000x^3y^2}{64x^7}} = \sqrt{\dfrac{125y^2}{8x^4}} = \dfrac{5\cdot|y|\sqrt{5}}{2x^2\cdot\sqrt{2}} = \dfrac{5\cdot|y|\sqrt{5}\cdot\sqrt{2}}{2x^2\cdot\sqrt{2}\cdot\sqrt{2}} = \dfrac{5\cdot|y|\cdot\sqrt{10}}{2x^2\cdot 2} = \dfrac{5\cdot|y|\cdot\sqrt{10}}{4x^2}$

b) $\dfrac{5\sqrt{2}+4\sqrt{3}}{6\sqrt{3}+7\sqrt{2}} = \dfrac{(5\sqrt{2}+4\sqrt{3})(6\sqrt{3}-7\sqrt{2})}{(6\sqrt{3}+7\sqrt{2})(6\sqrt{3}-7\sqrt{2})} = \dfrac{30\sqrt{6}-35\cdot 2+24\cdot 3-28\sqrt{6}}{36\cdot 3-49\cdot 2} = \dfrac{2+2\sqrt{6}}{10} = \dfrac{2(1+\sqrt{6})}{10} = \dfrac{1+\sqrt{6}}{5}$

$\dfrac{6\sqrt{10}-4\sqrt{15}}{2\sqrt{3}-3\sqrt{2}} = \dfrac{(6\sqrt{10}-4\sqrt{15})\cdot(2\sqrt{3}+3\sqrt{2})}{(2\sqrt{3}-3\sqrt{2})\cdot(2\sqrt{3}+3\sqrt{2})} = \dfrac{12\sqrt{30}+18\sqrt{20}-8\sqrt{45}-12\sqrt{30}}{4\cdot 3-9\cdot 2} = \dfrac{18\sqrt{4\cdot 5}-8\sqrt{9\cdot 5}}{-6} =$

$\dfrac{18\sqrt{4\cdot 5}-8\sqrt{9\cdot 5}}{-6} = \dfrac{36\sqrt{5}-24\sqrt{5}}{-6} = \dfrac{12\sqrt{5}}{-6} = -2\sqrt{5}$

$\dfrac{2\sqrt{6}+5\sqrt{3}}{3\sqrt{5}-4\sqrt{10}} = \dfrac{(2\sqrt{6}+5\sqrt{3})(3\sqrt{5}+4\sqrt{10})}{(3\sqrt{5}-4\sqrt{10})(3\sqrt{5}+4\sqrt{10})} = \dfrac{6\sqrt{30}+8\sqrt{60}+15\sqrt{15}+20\sqrt{30}}{9\cdot 5-16\cdot 10} = \dfrac{26\sqrt{30}+8\sqrt{60}+15\sqrt{15}}{-115}$

$\dfrac{9\sqrt{2a}-6}{2\sqrt{3a}+6\sqrt{3}} = \dfrac{(9\sqrt{2a}-6)\cdot(2\sqrt{3a}-6\sqrt{3})}{(2\sqrt{3a}+6\sqrt{3})\cdot(2\sqrt{3a}-6\sqrt{3})} = \dfrac{18a\sqrt{6}-54\sqrt{6a}-12\sqrt{3a}+36\sqrt{3}}{4\cdot 3a-36\cdot 3} =$

$\dfrac{18a\sqrt{6}-54\sqrt{6a}-12\sqrt{3a}+36\sqrt{3}}{12a-108} = \dfrac{6(3a\sqrt{6}-9\sqrt{6a}-2\sqrt{3a}+6\sqrt{3})}{6(2a-18)} = \dfrac{3a\sqrt{6}-9\sqrt{6a}-2\sqrt{3a}+6\sqrt{3}}{2a-18}$

$\dfrac{\sqrt{y}-\sqrt{x^3}}{\sqrt{y^3}+\sqrt{x}} = \dfrac{(\sqrt{y}-\sqrt{x^3})(\sqrt{y^3}-\sqrt{x})}{(\sqrt{y^3}+\sqrt{x})(\sqrt{y^3}-\sqrt{x})} = \dfrac{(\sqrt{y}-\sqrt{x^3})(\sqrt{y^3}-\sqrt{x})}{y^3-x} = \dfrac{y^2-\sqrt{xy}-xy\sqrt{xy}+x^2}{y^3-x}$

8a) $(2x + 3)^2 - (3x + 2)^2 = (2x + 3)(2x - 3) + 10$

$4x^2 + 12x + 9 - (9x^2 + 12x + 4) = 4x^2 - 9 + 10$

$4x^2 + 12x + 9 - 9x^2 - 12x - 4 = 4x^2 - 9 + 10$

$-5x^2 + 5 = 4x^2 + 1$

$4 = 9x^2$

$x^2 = \dfrac{4}{9}$

$\Leftrightarrow x = \dfrac{2}{3} \;\; \text{v} \;\; x = -\dfrac{2}{3}$

$IL = \{-\dfrac{2}{3} ; -\dfrac{2}{3}\}$

8b) $(x + 3)^2 + (x - 4)^2 - (x + 5)^2 = 2(x - 3)^2 - 18$

$x^2 + 6x + 9 + x^2 - 8x + 16 - (x^2 + 10x + 25) = 2(x^2 - 6x + 9) - 18$

$2x^2 - 2x + 25 - x^2 - 10x - 25 = 2x^2 - 12x + 18 - 18$

$x^2 - 12x = 2x^2 - 12x \;|\; +12x - x^2$

$0 = x^2 \qquad \text{v} \qquad x = 0$

$IL = \{0\}$

c) $(3x - 1)^2 - (2x + 2{,}5)^2 = 8x(x - 2) - 12$

$9x^2 - 6x + 1 - (4x^2 + 10x + 6{,}25) = 8x^2 - 16x - 12$

$9x^2 - 6x + 1 - 4x^2 - 10x - 6{,}25 = 8x^2 - 16x - 12$

$5x^2 - 16x - 5{,}25 = 8x^2 - 16x - 12$

$6{,}75 = 3x^2$

$2{,}25 = x^2$

$x = 1{,}5 \;\; \text{v} \;\; x = -1{,}5$

$IL = \{-1{,}5; 1{,}5\}$