

c.

$$f_t(x) = \frac{t + \ln(x)}{x} = \frac{t}{x} + \frac{\ln(x)}{x}$$

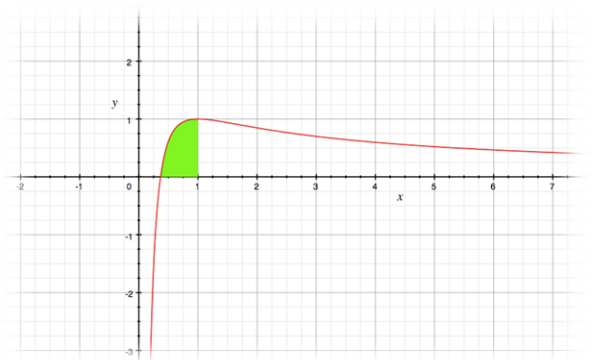
$$\int_{e^{-t}}^{e^{1-t}} \left( \frac{t}{x} + \frac{\ln(x)}{x} \right) dx = \left[ t \cdot \ln(x) + \frac{(\ln(x))^2}{x} \right]_{e^{-t}}^{e^{1-t}}$$

$$= \left[ \frac{2t \ln(x) + (\ln(x))^2}{2} \right]_{e^{-t}}^{e^{1-t}}$$

$$= \left( \frac{2t \cdot (1-t) + (1-t)^2}{2} \right) - \left( \frac{2t \cdot (-t) + (-t)^2}{2} \right)$$

$$= \left( \frac{2t - 2t^2 + 1 - 2t + t^2}{2} \right) + \left( \frac{2t^2 - t^2}{2} \right)$$

$$= \frac{1}{2}$$



Egal was man für  $t$  einsetzt, in den Grenzen von  $e^{-t}$  bis  $e^{1-t}$ , ergibt sich in allen Kurvenscharen die Fläche  $\frac{1}{2}$ .

d.

$$\pi \int_{e^{-2}}^h (f_2(x))^2 dx = \pi \int_{e^{-2}}^h \left( \frac{2 + \ln(x)}{x} \right)^2 dx$$

$$\pi \int_{e^{-2}}^h \left( \frac{4}{x^2} + \frac{4 \ln(x)}{x^2} + \frac{(\ln(x))^2}{x^2} \right) dx$$

$$\pi \left[ \frac{4}{x} - \frac{4 \ln(x) - 4}{x} - \frac{(\ln(x))^2 - 2 \ln(x) - 2}{x} \right]_{e^{-2}}^h$$

$$\pi \left[ \frac{-4 - 4 \ln(x) - 4 - (\ln(x))^2 - 2 \ln(x) - 2}{x} \right]_{e^{-2}}^h$$

$$\pi \left[ \frac{-10 - 6 \ln(x) - (\ln(x))^2}{x} \right]_{e^{-2}}^h$$