

c.

$$f_t(x) = \frac{t + \ln(x)}{x} = \frac{t}{x} + \frac{\ln(x)}{x}$$

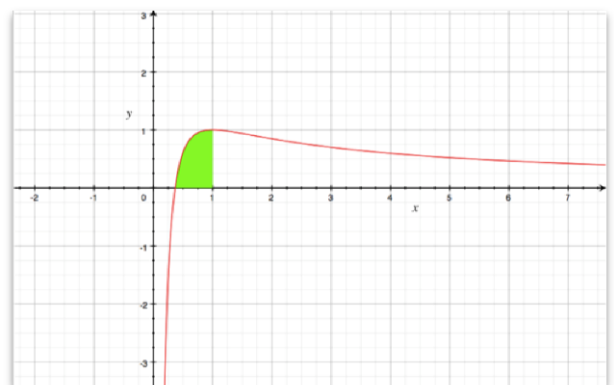
$$\int_{e^{-t}}^{e^{1-t}} \left(\frac{t}{x} + \frac{\ln(x)}{x} \right) dx = \left[t \cdot \ln(x) + \frac{(\ln(x))^2}{2} \right]_{e^{-t}}^{e^{1-t}}$$

$$= \left[\frac{2t \ln(x) + (\ln(x))^2}{2} \right]_{e^{-t}}^{e^{1-t}}$$

$$= \left(\frac{2t \cdot (1-t) + (1-t)^2}{2} \right) - \left(\frac{2t \cdot (-t) + (-t)^2}{2} \right)$$

$$= \left(\frac{2t - 2t^2 + 1 - 2t + t^2}{2} \right) + \left(\frac{2t^2 - t^2}{2} \right)$$

$$= \frac{1}{2}$$



Egal was man für t einsetzt, in den Grenzen von e^{-t} bis e^{1-t} , ergibt sich in allen Kurvenscharen die Fläche $\frac{1}{2}$.

d.

$$\pi \int_{e^{-2}}^h (f_2(x))^2 dx = \pi \int_{e^{-2}}^h \left(\frac{2 + \ln(x)}{x} \right)^2 dx$$

$$= \pi \int_{e^{-2}}^h \left(\frac{4}{x^2} + \frac{4 \ln(x)}{x^2} + \frac{(\ln(x))^2}{x^2} \right) dx$$

$$= \pi \left[-\frac{4}{x} - \frac{4 \ln(x) - 4}{x} - \frac{(\ln(x))^2 - 2 \ln(x) - 2}{x} \right]_{e^{-2}}^h$$

$$= \pi \left[\frac{-4 - 4 \ln(x) - 4 - (\ln(x))^2 - 2 \ln(x) - 2}{x} \right]_{e^{-2}}^h$$

$$= \pi \left[\frac{-10 - 6 \ln(x) - (\ln(x))^2}{x} \right]_{e^{-2}}^h$$